In the existing literature with a static model, the transfer paradox is derived by three logics. They are income decreasing, real wage decreasing, and externalities. The first one is that by decreasing wages and rents, the regional income of the transferor decreases (e.g., Johnson 1955, and Jones 1985). In this line, Beladi (1990) introduces unemployment and minimum real wage rate, Ohyama (1974) and Yano and Nugent (1999) assume tariff, and Bhagwati et al. (1983)
adopt a three-agent model. The second logic is that if the transfer increases both the income and the price of goods, then the transfer possibly decreases the real wage of the recipient (e.g., Bhagwati 1958). The final logic is presented in Naito (2003), which considers environmental effects in utility function, and suggests that the transfer can make the Pareto Improvement.


Johnson (1955), Mundell (1960), Jones (1975, 1985), Yano (1983), and Wang (1985) suggest that the transfer may increase the donor regional income by improving the terms-of-trade for the donor. In other words, the transfer increases the total demand of the exporting good in the donor. As a result, the donor’s income becomes higher by increasing the price and amount of the exporting good. These studies heavily depend on assumptions such as perfect competition market, and immobility of production factors. But, these assumptions do not fit in the globalizing economy. A New Economic Geography (NEG) framework incorporates with the increasing-returns-to-scale technology, monopolistic competition market, transport costs, and factor mobility.

The NEG framework is widely studied in the international trade and regional science literature. This is started by a series of papers of Krugman (1980, 1991), and is extended by the books of Fujita et al. (1999), Fujita and Thisse (2002), and Baldwin et al. (2003). There are two types of models called core-periphery and Home-market effect (HME) models in the NEG framework. In the core-periphery model, there are two countries having the same tastes, technologies, and size. The core-periphery model shows the sustain point and the break point. The sustain point is the critical value at which established agglomeration is no longer sustainable. The break point is the critical value at which symmetry equilibrium breaks. Meanwhile, the home market effect (Krugman 1980, Helpman and Krugman 1985) is that if other things are equal, a country with a larger home market has a more-than-proportionate manufacturing sector, and is a net exporter of the manufactured goods. The footloose capital model is used to study the HME. It supposes that two regions differ only in size. In addition, residents own capital and obtain capital rent from investment in firms as fixed inputs.

Brakman et al. (2007) examine the unilateral transfer effect in the core-periphery model, and conclude that the transfer paradox cannot be observed in this model even if a bystander is present. Takarada (1998) considers the transfer problem in a context allowing international capital mobility. He concludes that the capital movement restrains the price change, and changes the welfare in reverse to the direction of improvement in terms-of-trade.

Different from their studies, this paper presents a new logic deriving the transfer paradox by use of a footloose capital model. We show that the transfer paradox is observable when we have an internationally mobile factor. Such a transfer paradox results from the change in capital revenue. Two more conditions are necessary. The income elasticity of demand is necessary to be larger than one and there is an inferior good. Those conditions are neglected in the literature. We construct a footloose capital model in the NEG framework, assuming that North provides an international income transfer to South.

There are some empirical studies supporting this logic. Rothman et al. (1994) tell us that the income elasticity of demand for manufactured goods ranges from 1.12 to 1.51 with a mean of 1.26, and the income elasticity of demand for grains and starches ranges from 0.06 to 0.45 with a mean of 0.19. They suggest that the income elasticity of demand for grains and starches might be minus.

This paper is organized as follows. In section 2, we establish the footloose capital model. In section 3, we examine the equilibrium and obtain the conditions for the transfer paradox.
In section 4, we provide conclusion.

2 The model

In this section, we use an NEG model, named footloose capital model of Martin and Rogers (1995). The economy space consists of two countries which are called North and South. North gives an income transfer $T$ to South, and South uses the transferred income to consume manufactured and agricultural goods. For the sake of simplicity, we assume that the transfer is collected and distributed in each country in lump sum non-distortionary forms. In the global economy, there are $L$ units of labor and $K$ units of capital. The labor share in North is $\theta_L$, the capital share in North is $\theta_K$, where $0 \leq \theta_L, \theta_K \leq 1$.

There are two sectors. The manufacturing sector is with monopolistic competition, increasing returns to scale, and positive transport costs. The agricultural sector is with perfect competition, constant returns to scale, and free transportation. A unit of labor products one unit of the homogeneous agricultural good. We choose the agricultural good as *numéraire*, so its price is $p^A = 1$. According to the free transportation assumption of the agricultural good, the wages in two countries are the same, $w_n = w_s = 1$. The population in each country is sufficiently larger than the labor demand for the agricultural good. Producing the manufactured goods requires labor as marginal cost and capital as fixed cost.

Next, we assume consumer behavior. As done in Johnson (1955), Jones (1985), and Yano and Nugent (1999), we describe the transfer paradox by the nominal income. In other words, we do not define the transfer paradox by utility or the real income. Different from the NEG literature which defines demand functions from utility functions, we assume the following demands directly.

\[
D^M_{nn} = \left(\frac{p_{nn}}{P_n}\right)^{-\sigma} a(Y_n)^{\alpha}, \quad D^M_{sn} = \left(\frac{p_{sn}}{P_n}\right)^{-\sigma} a(Y_n)^{\alpha},
\]

(1)

\[
D^M_{ss} = \left(\frac{p_{ss}}{P_s}\right)^{-\sigma} b(Y_s)^{\beta}, \quad D^M_{ns} = \left(\frac{p_{ns}}{P_s}\right)^{-\sigma} b(Y_s)^{\beta},
\]

(2)

\[
D^A_n = Y_n - a(Y_n)^{\alpha}, \quad D^A_s = Y_s - b(Y_s)^{\beta},
\]

(3)

where $D^k_{ij}$ is a region $j$’s demand of $k$ type sector made in region $i$, $Y_i$ is the regional income in country $i$, $p_{ij}$ is region $j$’s price of a manufactured good made in region $i$, $P_i$ is the price index of manufactured goods in region $i$, $a, b \geq 0$ are constant parameters, and $\alpha$ and $\beta$ are the income elasticity of demand for manufactured goods in each country. Demand functions are calculated from utility functions in NEG models, such as the Cobb-Douglas utility function (Fujita et al. 1999) and quasi-linear utility function (Ottaviano et al. 2002). With respect to the Cobb-Douglas function, the income elasticity of demand is one, $\alpha = \beta = 1$, and $a$ and $b$ are the expenditure shares of manufactured goods. As will be seen later, such utility functions fail to observe the transfer paradox.

The consumptions of manufactured and agricultural goods are non-negative (i.e., $D^k_{ij} \geq 0$). We assume a composite good $M$ which is made up of a number of differentiated varieties $i$

\[
M_n = \left[ \int_0^N D^M_n (i) \frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma}{\sigma - 1}},
\]

(4)

where $\sigma > 1$ represents the elasticity of substitution between two manufactured varieties, $N$ is the number of varieties, and $D^M_n$ is the demand of a typical manufactured good $i$ in North.
The manufacturing price index is given by

$$P_n = \left[ \int_0^N p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}. \quad (5)$$

Although we do not assume the Cobb-Douglas utility function, we can use the price index without knowing the upper level of demand function (Fujita et al. 1999 pp.46-47).

Then, we describe the production in the manufacturing sector. We assume that a fixed input of one unit of capital and a marginal input of \((\sigma - 1)/\sigma\) units of labors are required. The firm share in North is equal to the capital share \(\lambda\), so \(N_n = \lambda K\) and \(N_s = (1 - \lambda)K\). Assume Samuelson’s iceberg international transportation costs. Specifically, \(\tau > 1\) units of a manufactured good must be shipped for one unit to reach the other country. Then, a particular firm located in North sets prices to maximize its profit

$$\max_{p_{nn}, p_{ns}} \pi_n = p_{nn} D_{nn}^M + p_{ns} D_{ns}^M - \frac{\sigma - 1}{\sigma} (D_{nn}^M + \tau D_{ns}^M) - r_n. \quad (6)$$

The first-order condition gives \(p_{nn} = 1\) and \(p_{ns} = \tau\). Therefore, the price indices are simplified as

$$P_n = \left[ (\lambda + \phi(1 - \lambda))K \right]^{\frac{1}{1-\sigma}}, \quad P_s = \left[ (\phi \lambda + (1 - \lambda))K \right]^{\frac{1}{1-\sigma}}, \quad (7)$$

where \(\phi \equiv \tau^{1-\sigma} \in (0, 1)\) is the trade freeness.

By the free-entry condition of firms, the profit of firms is zero in a long-run equilibrium. Then, we obtain the capital rent as below

$$r_n = \frac{1}{\sigma K} \left[ \frac{a(Y_n)^\alpha}{\lambda + \phi(1 - \lambda)} + \frac{\phi b(Y_s)^\beta}{\phi \lambda + (1 - \lambda)} \right], \quad (8)$$

$$r_s = \frac{1}{\sigma K} \left[ \frac{\phi a(Y_n)^\alpha}{\lambda + \phi(1 - \lambda)} + \frac{b(Y_s)^\beta}{\phi \lambda + (1 - \lambda)} \right]. \quad (9)$$

The regional incomes are

$$Y_n = \theta_K K(\lambda r_n + (1 - \lambda)r_s) + \theta_L L - T, \quad (10)$$

$$Y_s = (1 - \theta_K) K(\lambda r_n + (1 - \lambda)r_s) + (1 - \theta_L) L + T. \quad (11)$$

3 Equilibrium

To study the equilibrium and its stability, we assume that markets adjust instantaneously. The international capital movement is relatively slow. We apply the following ad hoc dynamic system to describe the international capital movement.

$$\dot{\lambda} = (r_n - r_s)(1 - \lambda) \lambda \quad (12)$$

According to Tabuchi and Zeng (2004), the equilibrium stability does not depend on this specific form in a two-country model.

3.1 Pareto Improvement

To obtain the conditions of the transfer paradox, we calculate the equilibrium and the differential of the regional income with respect to \(T\). Let \(m_i \equiv P_i \frac{\partial M}{\partial Y_i}\) be the marginal propensity for
the composite manufactured goods in region \( i \). Then

\[
\frac{dY_n}{dT} = \frac{\theta_K(m_s - m_n)}{\sigma - (\theta_K m_n + (1 - \theta_K)m_s)} - 1
\tag{13}
\]

\[
\frac{dY_s}{dT} = \frac{(1 - \theta_K)(m_s - m_n)}{\sigma - (\theta_K m_n + (1 - \theta_K)m_s)} + 1
\tag{14}
\]

Proof see Appendix A.

The first term of the right hand sides of equations (13) and (14) are the indirect effect of income transfer, which are the change of capital rent. The second term of right hand sides of equation (13) and (14) are the direct effect of income transfer.

From equation (13), we obtain conditions for a Pareto Improvement. Specifically, \( \frac{dY_n}{dT} > 0 \) if and only if either of following conditions holds.

\[
m_n < \sigma < m_s \quad \text{and} \quad \theta_K m_n + (1 - \theta_K)m_s < \sigma
\tag{15}
\]

\[
m_s < \sigma < m_n \quad \text{and} \quad \theta_K m_n + (1 - \theta_K)m_s > \sigma
\tag{16}
\]

Equations (15) and (16) indicate that the transfer paradox is not observable if \( \theta_K = 0 \). It means that having capital in North plays an important role to observe the transfer paradox. Empirically, most donors are richer and have a large share of capital. Therefore a positive \( \theta_K \) is quite likely. Meanwhile, it is difficult to satisfy equation (16) in numerical simulation (See Appendix B). As a result, equation (15) is the only condition for the income transfer to make North income better off.

The second condition of equations (15) can be understood as below,

\[
K \frac{d r^*}{dT} = \frac{m_s - m_n}{\sigma - (\theta_K m_n + (1 - \theta_K)m_s)}.
\]

It is the derivative of the total capital revenue with respect to \( T \) (See Appendix A). If equation (15) is satisfied, \( \frac{d r^*}{dT} \) is always positive. Therefore,

\[
\frac{dY_s}{dT} = (1 - \theta_K)K \frac{d r^*}{dT} + 1 > 0
\]

holds. As a result, if equation (15) is satisfied, the transfer results in a Pareto Improvement.

We know that the sum of the marginal propensity for the composite manufactured good \( m_i^M \) and the marginal propensity for the agricultural good \( m_i^A \) is always one, \( m_i^A + m_i^M = 1 \). Therefore, the consumption of the agricultural good in South decreases by the income transfer (\( m_s^A < 0 \)) if \( m_s > \sigma > 1 \). In other words, the agricultural good is inferior in South.

The intuition of equation (15) is given below. If the transferee decreases the agricultural good consumption and consumes manufactured goods more than the income transfer, and besides, the decrement of manufactured goods consumption in the transferor is smaller, then the total demand of manufactured goods and the capital return increase. If the increment of the capital return is larger than the transfer, the paradox is observed.

There are some empirical studies supporting this logic. For example, Seale et al. (1991) estimate the income elasticity of demand for energy, which is found to be larger than one in 51 countries. Rothman et al. (1994) examine prices of goods and expenditure in 53 countries contains including both developed countries and developing countries. They find that the income elasticities of demand for energy, electricity, health, education, and other goods are all larger than one, while the income elasticity of demand for grains and starches is small. Quite interestingly, the income elasticity of demand for grains and starches might be negative.
3.2 Pareto Inferior

From equation (14), we obtain the following conditions for the recipient’s income to be decreased by an income transfer. Specifically,

\[ \frac{dY_s}{dT} < 0 \text{ holds iff either of the following conditions holds.} \]

\[ m_s < \sigma < m_n \quad \text{and} \quad \theta_K m_n + (1 - \theta_K) m_s < \sigma \]  \hspace{1cm} (17)

\[ m_n < \sigma < m_s \quad \text{and} \quad \theta_K m_n + (1 - \theta_K) m_s > \sigma \]  \hspace{1cm} (18)

Equation (17) is the reverse of equation (16) while equation (18) is the reverse of equation (15). Equations (17) and (18) indicate that the transfer paradox is not observable if \( \theta_K = 1 \). Equation (18) seems to be impossible in numerical simulation (See Appendix B), therefore, the condition is given by equation (17) only. In this situation, the agricultural good is inferior in North because of the marginal propensity to the manufactured goods is larger than one. In other words, the agricultural consumption in North increases by an income transfer \( (m^A_n < 0) \).

If equation (17) is satisfied, the derivative of the total capital rent is always negative. Therefore,

\[ \frac{dY_n}{dT} = \theta_K K \frac{dr^*}{dT} - 1 < 0 \]

holds.

The intuition of equation (17) is given below. If the transferor greatly decreases the manufactured goods consumption in North, while the transferee slightly increases the manufactured goods consumption in South, then the capital rent decreases because the total demand of the manufactured goods decreases. If the decrement of the capital return is larger than the direct effect of the transfer in the transferee, the paradox is observed.

**Proposition 1** When \( \theta_K m_n + (1 - \theta_K) m_s < \sigma \), an international income transfer increases both regional incomes (i.e., Pareto Improvement) if \( m_n < \sigma < m_s \) holds, and an international income transfer decreases regional income of the transferee if \( m_s < \sigma < m_n \) holds.

In this model, the transfer paradox is defined by regional income. The paradox occurs due to the capital rent change. From the proposition, it is important that the income elasticity of demand in one country is larger than \( \sigma \), and there are inferior good.

The idea of an inferior good is discussed in Johnson (1967) and Brecher and Bhagwati (1982). They assume that if the transferee increases the consumption of the inferior good, the transferee utility will decrease. Although our logic is different from theirs, the result here is consistent with their papers because we do not define the transfer paradox with respect to utility. Moreover, the consumption of the agricultural good in South as an inferior good decreases by the transfer in equation (15), and the consumption of the agricultural good as an inferior in North increases in equation (17).

We know that the transfer paradox conditions are related to the marginal propensity for manufactured goods \( m_i \) and the elasticity \( \sigma \) of substitution between any two varieties. But, Hanson (2005) estimates that \( \sigma \) ranges between 4.9 and 7.6 in the nonlinear least estimation and between 1.7 and 2.5 in the generalized method of moments. Brakman et al. (2004) estimate \( \sigma \) is between 3.1 and 5.0. Head and Ries (2001) estimate \( \sigma \) is between 7.9 and 11.4. Feenstra (1994) checks the various manufactured goods and estimates that \( \sigma \) ranges between 3.0 and 8.4. Therefore, it is rare for the marginal propensity to be larger than \( \sigma \) in the real world, which explains that we seldomly observe such a transfer paradox.

4 Conclusion

In this paper, we analyze the transfer paradox in an NEG model. In the section 2, we construct a footloose capital model, in which the demands of consumers are given directly rather than
derived from a utility function. In the section 3, we obtain the conditions for the transfer paradox.

In the new logic of the transfer paradox in this paper, the capital and the marginal propensity for the manufactured goods play very important roles. The difference of the marginal propensity brings a change in the total demand, and then in the capital rent. If this change of the capital rent is larger than the direct effect of the transfer, the international income transfer makes both countries better/worse off. From the conditions of the transfer paradox, we find that three facts are important. The first is the existence of capital in both countries. The second is the difference of the marginal propensities. The final one is that the income elasticity of demand is larger than one (i.e., inferior good).

Appendix A

We solve the model and obtain equations (13) and (14). The firms share in an interior equilibrium from the assumption of free capital movement \((r_n = r_s)\), is

\[
\lambda = \frac{a(Y_n)^\alpha - \phi b(Y_s)^\beta}{(1 - \phi)(a(Y_n)^\alpha + b(Y_s)^\beta)}.
\]

Then, the equilibrium capital rent is

\[
r^* = \frac{a(Y_n)^\alpha + b(Y_s)^\beta}{\sigma K}.
\]

Therefore, the regional incomes are given by

\[
Y_n = \frac{\theta_K(a(Y_n)^\alpha + b(Y_s)^\beta)}{\sigma} + \theta_L L - T, \tag{19}
\]

\[
Y_s = \frac{(1 - \theta_K)(a(Y_n)^\alpha + b(Y_s)^\beta)}{\sigma} + (1 - \theta_L)L + T. \tag{20}
\]

Finally, these derivatives of regional incomes with respect to \(T\) are given by equations (13) and (14).

Moreover, the derivative of the capital rent with respect to \(T\) are given by

\[
\frac{dr^*}{dT} = \frac{m_s - m_n}{K(\sigma - (\theta_K m_n + (1 - \theta_K)m_s))}.
\]

Appendix B

With respect to equations (16) and (18), we need to consider existence. It is difficult to solve equations (19) and (20), but we obtain the following result.

Let

\[
Z \equiv Y_n - \left[ \frac{a(Y_n)^\alpha + b(Y_s)^\beta}{\sigma} + \theta_L L - T \right],
\]

which is plotted in Figure 1. It is the difference between \(Y_n\) and \(\frac{a(Y_n)^\alpha + b(Y_s)^\beta}{\sigma} + \theta_L L - T\), when \(\theta_K = 1\). The horizontal axis is \(Y_n\), and the vertical axis is \(Z\). The figure depicts an example of equation (19). The parameters in this figure are given by \(L = 100, \theta_L = 0.6, \sigma = 2, \beta = 1, b = 0.3\). If the line is not tangent to or does not cross the horizontal axis, there is no solution. Moreover, if the slope is always negative, neither the solution exists. Therefore, we have

\[
\frac{dZ}{dY_n} = 1 - \frac{a\alpha(Y_n)^{\alpha - 1}}{\sigma} < 0
\]

which implies that \(\sigma < m_n\). Thus, equation (16) is false.

Similarly, equation (20) has no solution, so equation (16) and (18) are false.
Figure 1: The solution of equation (19)

References


